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## The Effect of Magnetic field on Boundary Layer Flow of a Viscoelastic Fluid Past a Nonlinear Stretching Sheet

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In the present article, we study the effect of magnetic field on the boundary layer flow of an electrically conducting viscoelastic fluid over a nonlinear stretching sheet. The governing boundary layer equations for the momentum and energy are transformed into the set of non-linear ordinary differential equations by using suitable similarity solutions. The transformed equations are then solved numerically through Keller-box method. The influence of nonlinear stretching sheet parameter, viscoelastic parameter and magnetic field parameter on the velocity and temperature fields are studied graphically. The effects of the skin friction coefficient and local Nusselt number was also studied and its observed that skin friction coefficient increases while the local Nusselt number decreases for higher value of nonlinear stretching parameter. It is also noted that the magnetic field have highest impacts on the temperature distributions as compared to the velocity field.

**Keywords:** Viscoelastic fluid, Magnetic field, Non-linear stretching sheet, Skin friction and Similarity solution

### 1. Introduction

The fluid flow over a stretching sheet is important in many practical applications such as extrusion of plastic sheets, paper production, glass blowing, metal spinning and polymers. In metal spring processes, the continuous casting of metals, drawing plastic films and spinning of fibres involves some aspects of flow over a stretching sheet. Thus, the desired characteristics of the final product strictly depends on the rate of cooling and the process of stretching the sheet. This phenomenon is influenced by the nature of the fluid layer adjacent to the stretching surface. Crane [1] studied the boundary layer flow over a linearly stretching sheet in an ambient fluid and gave a similarity solution in a closed analytical form for the steady two-dimensional problem. Ishak et al. [2] analytically investigated the unsteady mixed convection boundary layer flow and heat transfer due to a stretching vertical sheet.

These investigations have a definite bearing on the problem of a polymer sheet extruded continuously from a dye. It is often assumed that the sheet is inextensible, but situations may arise in the polymer industry in which it is necessary to deal with a stretching of a plastic

sheet. In view of this, Vajravelu [3] presented in his article that, the stretching velocity of the sheet may not necessary be in linear form and hence studied the viscous fluid and heat transfer of two dimensional boundary layer flow over a nonlinear stretching sheet in the absence of viscous dissipation in the energy equation. His result showed that an increased in nonlinear stretching parameter leads to an increased in wall shear stress and that the transport of heat is often from the surface of the sheet.

Cortell [4] studied the effects of viscous dissipation and variable surface temperature in the fluid flow over a nonlinear stretching sheet. However, the transport phenomenon in an incompressible Micropolar fluid flow over a nonlinear stretching sheet was examined by Bhargava et al. [5] with the help of two numerical techniques, i.e. the finite element method (FEM) and finite difference method (FDM). Their results showed that an angular velocity rises with the increased in nonlinear sheet parameter and reduced with an increased in convective parameter. Hamad and Ferdows [6] discussed the similarity solution of the boundary layer flow and heat

transfer in nanofluid past a nonlinear stretching sheet. Handy et al. [7] also addressed the effect of radiation on viscous nanofluid flow and heat transfer over a nonlinear stretching sheet. Khaddar and Megahed [8] analyzed the boundary layer flow past a nonlinear stretching sheet with variable thickness and slip condition by using finite difference method (FDM). Mabood et al. [9] also obtained a numerical study of MHD boundary layer flow and heat transfer of an electrically conducting water based nanofluid over a nonlinear stretching sheet.

Hayat et al. [10] discussed and elaborated the magnetohydrodynamics nonlinear convective flow of Walter-B nanofluid over a nonlinear stretching sheet with variable thickness. Madikare et al. [11] studied the boundary layer flow and heat transfer of Casson fluid generated due to nonlinear stretching sheet in the presence of viscous dissipation. On the other hand, Kazemi et al. [12] analytically solved the viscous fluid flow and heat transfer past a nonlinear stretching sheet by considering two cases of boundary conditions, i.e. Prescribed Heat Flux (PHF) and of Prescribed Surface Temperature (PST) using Homotopy analysis method (HAM).

One of the mechanisms that influences the behaviour and performance of fluid flow is the presence of magnetic field in the flow problem. The concept is termed as "magnetohydrodynamics (MHD) fluid flow. An example of this kind of fluids are electrolyte or saltwater, liquid metal and plasma. The study of MHD boundary layer flow past a stretching sheet has many geothermal and industrial applications. For examples, cooling of nuclear reactors, liquid metal fluids, high temperature plasmas, MHD power generators and MHD accelerators. These numerous applications prompt many researchers into the study of MHD flows with different fluids, geometry and boundary conditions [13-17].

Non-Newtonian fluids have developed a lot interests by many researchers due to its practical applications in biological materials (animal blood and synovial liquids), chemical materials (polymer liquids, detergent, paints and shampoo) and food processing (condensed milk, ketchup and chocolates).

Although, there is no single constitutive equations that exhibit all the rheology of these fluids due to its complexity in nature. Hence, different models have been developed to describe the rheological behaviour of these fluids. One of the most interesting non-Newtonian fluids is viscoelastic fluid. In most of the polymer processing applications one deals with flows of a viscoelastic fluid over a stretching sheet. The study of boundary layer flow of viscoelastic fluids has developed a considerable attention in the last decades due to its numerous applications in many branches of science, technology and engineering, particularly in geophysics, material processing, bioengineering, chemical and nuclear industries. Viscoelastic fluid also plays a vital role in the manufacturing process, such as in the production of paints, inks, coatings, papers etc. [18]. In view of this, Rajagopal et al. [19] studied the flow behavior of a viscoelastic fluid over a stretching sheet and obtained similarity solutions of the boundary layer equations numerically for the case of small viscoelastic parameter  $k_1$ . They reported that that skin-friction decreases with increase in  $k_1$ . Similarly, the heat and mass transfer effects in a boundary layer flow through porous medium of an electrically conducting viscoelastic fluid subject to transverse magnetic field in the presence of heat source/sink and chemical reaction have been analyzed by Nayak et al [20]. Mishra et al. [21] studied the effects of transverse magnetic field on an electrically conducting viscoelastic (Walters  $B'$ ) fluid over a stretching surface in presence of non-uniform heat source. The inclusion of magnetic field is counterproductive in diminishing the velocity distribution whereas reverse effect is encountered for temperature distribution. However, the two-dimensional boundary layer flow problem and heat transfer characteristic of ferromagnetic viscoelastic fluid flow over a stretching surface with a linear velocity under the impact of magnetic dipole and suction has studied by Majeed et al. [22]. They reported from their results that pressure profile and skin friction coefficient increase with the variation of ferromagnetic interaction parameter. An analytical investigation on the entropy generation effect for viscoelastic fluid flow involving inclined magnetic field and non-linear

thermal radiation with heat source and sink over a stretching sheet has studied by Hakeem et al. [23]. It was revealed from their results that the existence of radiation and heat source parameters would reduce the entropy production and at the same time, aligned magnetic field and viscoelastic parameters would produce more entropy.

Most of work of the viscoelastic flow over a stretching sheet in the literature considers linear stretching sheet. Although, recently Mustapha [24] addressed the analytical solution of a viscoelastic fluid flow past a nonlinear stretching surface using optimal Homotopy analysis method. However, numerical solution of MHD viscoelastic fluid flow and heat transfer over a nonlinear stretching sheet have not yet reported in the literature. In this article, we studied the influence of magnetic field on boundary layer flow and heat transfer of viscoelastic fluid past a nonlinear stretching sheet. The equations that governed the fluid flow and heat transfer problem are solved numerically using Keller-box method. The impact of pertinent parameters like nonlinear stretching sheet, viscoelastic and magnetic field parameters on velocity field and temperature distributions are depicted graphically and analyzed..

## 2. Mathematical Formulation

We consider the steady two-dimensional incompressible boundary layer flow of viscoelastic fluid induced by a nonlinear stretching sheet. The sheet is stretched along  $x$ -axis with the velocity  $U_w = ax^{2n-1}$ , where  $n$  is the stretching sheet parameter and  $a$  is constant. We shall also assume a variable magnetic field of strength  $B(x) = B_0x^{2n-1}$  applied normal to the direction of fluid flow, where  $n$  is the power law index parameter,  $B_0$  is the magnetic field constant. The two-dimensional continuity, momentum and energy equations that governed the viscoelastic fluid flow problem is given by, (see Beard and Walters [25])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} - \frac{k_0}{\rho} \left( u \frac{\partial^3 u}{\partial x \partial y^2} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} + v \frac{\partial^3 u}{\partial y^3} \right) - \frac{\sigma}{\rho} B^2(x)u \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the following boundary conditions

$$u = U_w(x), \quad v = 0, \quad T = T_w = T_\infty + bx^{2n-1} \text{ at } y = 0 \tag{4}$$

$$u \rightarrow 0, \quad \frac{\partial u}{\partial y} \rightarrow 0, \quad T \rightarrow 0 \text{ as } y \rightarrow \infty$$

where  $u$ , denote the horizontal velocity and  $v$ , the vertical velocity in  $xy$  – direction respectively,  $\nu$  kinematic viscosity,  $\rho$  fluid density,  $\sigma$  is the electrical conductivity,  $T$  temperature,  $\alpha$  is the thermal diffusivity. The temperature at the wall is defined as  $T_w = T_\infty + bx^{2n-1}$ , where  $b$  is constant and  $T_\infty$  is the free stream temperature.

We introduce the following similarity variables in order to reduces the governing partial differential equations into system of ordinary differential equations,

$$\eta = \sqrt{\frac{(n+1)u}{2\nu x}} y, \quad \psi(x, y) = \sqrt{\frac{2\nu ux}{(n+1)}} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{5}$$

where  $\psi(x, y)$  represents the stream function and is defined as

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{6}$$

Substituting Eqs. (5) and (6) into Eqs. (1-3) yields a dimensionless ordinary differential equation,

$$f''' + \frac{2n}{n+1} f f'' - \frac{2(2n-1)}{n+1} f'^2 + K \{ n f''^2 - 2(3n-2) f' f''' + n f f^{(4)} \} - \left( \frac{2}{n+1} \right) M f' = 0 \tag{7}$$

$$\theta'' - \left( \frac{2}{n+1} \right) Pr \{ (2n-1) f' \theta + n f \theta' \} = 0. \tag{8}$$

The boundary conditions (4) then becomes

$$f(\eta) = 0, \quad f'(\eta) = 1, \quad \theta(\eta) = 1 \text{ at } \eta = 0 \tag{9}$$

$$f'(\eta) = 0, \quad f''(\eta) = 0, \quad \theta(\eta) = 0 \text{ as } \eta \rightarrow \infty.$$

The primes represent the derivative of  $f$  with respect to  $\eta$ ,  $K = \frac{k_0 U_w}{\rho \nu x}$  is the viscoelastic parameter,

$M = \frac{\sigma B_0^2}{\rho \alpha}$  is the magnetic field parameter,  $Pr = \frac{\nu}{\alpha}$  is the Prandtl number. The physical parameters of principal interests are the coefficient of skin friction and local Nusselt number, which are defined as

$$C_f = \frac{\tau_w}{\rho u_w^2} \quad \text{and} \quad Nu_x = \frac{x q_w}{\alpha(T_w - T_\infty)}, \quad (10)$$

where  $\tau_w$  is the skin friction at the wall and  $q_w$  is the heat flux at the surface.

### 3. Method of Solution

The sets of coupled nonlinear differential equations (1-3) and the boundary conditions (4) is reduced to dimensionless nonlinear ordinary differential equations (7-9) with the help of similarity transformation (5). An effective numerical scheme method know as Keller-box method is then used to solve the nonlinear ordinary differential equations (7-8) and the boundary condition (9). The details explanations of this numerical method are contained in the book of Cebeci and Bradshaw [26].

### 4. Results and Discussion

In this paper, the study of an electrically conducting viscoelastic fluid and heat transfer over a non-linear stretching sheet is

**Table 1.** Comparison of skin friction coefficient  $-f''(0)$  and local Nusselt number  $-\theta'(0)$  with different values of  $n$  for  $M = K = 0$  and  $Pr = 0.71$ .

$n$	$-f''(0)$		$-\theta'(0)$	
	Vajravelu [3]	Present results	Vajravelu [3]	Present results
0	1.0000	0.9998	0.4590	0.4591
5	1.1945	1.1946	0.4394	0.4391
10	1.2348	1.2346	0.4357	0.4357

**Table 2.** Comparison of skin friction coefficient and local Nusselt number for various physical parameters

$n$	$M$	$K$	$Pr$	$-f''(0)$	$-\theta'(0)$
1	2	1	0.7	3.1145	0.7542
3	2	1	0.7	3.7984	0.5548
5	2	1	0.7	4.3876	0.3651
3	0	1	0.7	0.7339	3.1003
3	2	1	0.7	1.3659	2.7835
3	4	1	0.7	1.7852	2.1285
3	2	0.1	0.7	1.2276	1.5643
3	2	0.5	0.7	0.8267	1.7690

considered. Comprehensive numerical computations are conducted for various values of physical parameters that govern the flow problem. The effects of non-linear stretching parameter  $n$ , viscoelastic parameter  $K$  and magnetic parameter  $M$  on velocity and temperature are illustrated graphically. In order to verify the accuracy of the present method, we have compared our results for skin friction coefficient  $-f''(0)$  and local Nusselt number  $-\theta'(0)$  with that of Vajravelu [3] for different values of  $n$  which are presented in Table 1. It is seen from this table that the present results coincide with his results, which confirm that the numerical method used in this paper, is perfect and accurate. The computed values of  $-f''(0)$  and  $-\theta'(0)$  are presented in Table 2 for different values of  $n$ ,  $K$ ,  $M$  and  $Pr$ . It is interesting to note that an increase in the values of  $n$  results in rapid increase in the magnitude of coefficient of skin friction  $-f''(0)$  and decrease in the value of local Nusselt number  $-\theta'(0)$ . On the other hand, an increase in the value of  $M$  results to the increased in the coefficient of skin friction and the reduction in the Nusselt number. Similarly, augmenting the values of  $K$  and  $Pr$  leads to enhancement in the rate of heat transfer.

3	2	1	0.7	0.6382	1.9943
3	2	1	1	0.8754	0.8639
3	2	1	2	0.8754	1.9267
3	2	1	5	0.8754	2.7183

Figures 1- 4 exhibit the effects of nonlinear stretching parameter  $n$  on velocity profile in the presence and absence of magnetic field. It is observed from these figures that the dimensionless velocity decreases with an increase in the value of nonlinear stretching sheet parameter  $n$  as shown in Figures 1 and 2. Thus, the decrease is higher in the presence of magnetic field. Although, different trends occurred for  $K = 0$ , which corresponds to viscous flow and it is relatively interesting to note the changes of velocity profile near  $\eta = 1.6$  as shown in Figure 3. Hence, the dimensionless velocity decreases with an increase in  $n$  for  $\eta < 1.6$  and increases with an increase in  $n$  for  $\eta > 1.6$ . An asymptotically decreasing smooth profile is noticed with the presence of magnetic field as shown in Figure 4. This shows the influence of magnetic field in the fluid flow problem.

Figures 5 – 8 show the effects of nonlinear stretching parameter  $n$  on temperature profile in the presence and absence of magnetic field. These figures show that the dimensionless temperature decreases with an increase in the value of nonlinear stretching sheet parameter  $n$  for a small value of viscoelastic parameter  $K$ , but the reverse is the case when the value of  $K$  is large as shown in Figures 7 and 8.

Figures 9 and 10 illustrate the effect of nonlinear stretching sheet parameter on temperature profile in the presence and absence of magnetic field. It is observed that there is an increase in the temperature distributions for an increase in  $n$  without the magnetic field and decrease in the temperature profile with the presence of magnetic field as shown in Figure 10. It is therefore evident that the presence of magnetic field in viscous fluid flow decreases the dimensionless temperature rapidly compare to viscoelastic fluid flow with respect to an increase in nonlinear stretching sheet parameter  $n$ . From these figures, we can deduce that, magnetic field

has highest impacts on temperature profile compare to velocity profile, especially for a small value of  $K$ .

Figures 11 -16 show the variation of viscoelastic parameter  $K$  on velocity and temperature profiles with respect to various values of nonlinearly stretching and magnetic field parameters  $n$  and  $M$  respectively. These figures depict an increase in velocity profile as the value of viscoelastic parameter increases as shown in Figure 11, which corresponds to linear stretching sheet. However, different trends were observed for nonlinear stretching sheet (i.e.  $n > 1$ ) whereby the dimensionless velocity decreases with an increase in viscoelastic parameter as shown in Figure 14. For the dimensionless temperature, there is a decrease in the temperature profile as the value of viscoelastic parameter increases without magnetic field and increases in the presence of magnetic field as shown in Figures 15 and 16 respectively.

Figure 17 - 19 presents the effects of magnetic field on velocity and temperature profiles. It is noted that as the value of  $M$  increases the velocity profile decreases for a small value of  $K$  and  $n$  (see Figure 17) and decreases rapidly for a larger value of  $K$  and  $n$  (see Figure 18). An interesting point is to note that the effect of magnetic field is to increase the temperature at all points which is a natural consequences of resistive force offered by magnetic field opposing the motion of the fluid, thereby, enhancing the thermal energy as a result of which temperature increases in the thermal boundary layer region as shown in Figure 19

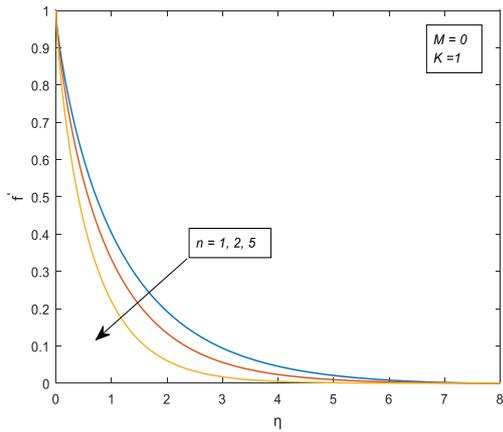


Figure 1. Variation of velocity profile  $f'$  for different values of  $n$  with  $M = 0$  and  $K = 1$

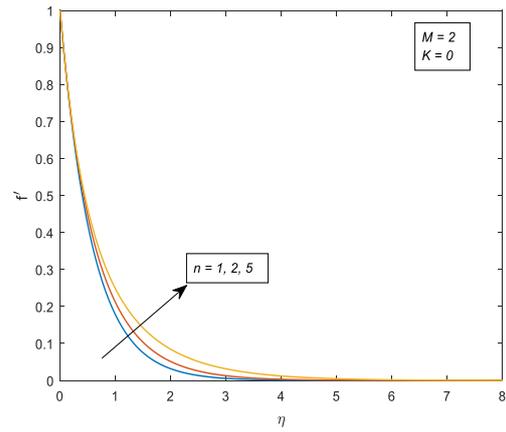


Figure 4. Variation of velocity profile  $f'$  for different values of  $n$  with  $M = 2$  and  $K = 0$

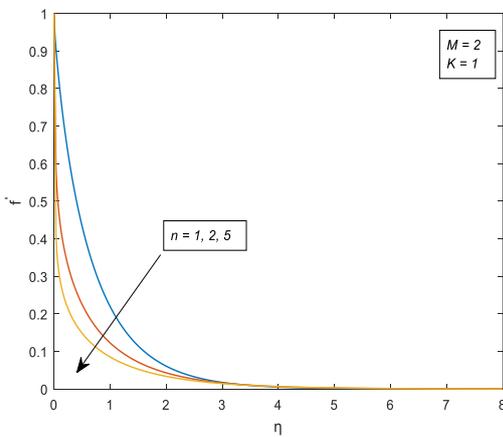


Figure 2. Variation of velocity profile  $f'$  for different values of  $n$  with  $M = 2$  and  $K = 1$

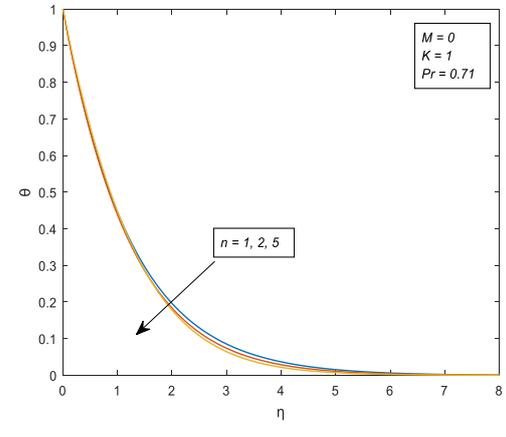


Figure 5. Variation of temperature profile  $\theta$  for different values of  $n$  with  $M = 0$ ,  $K = 1$  and  $Pr = 0.71$

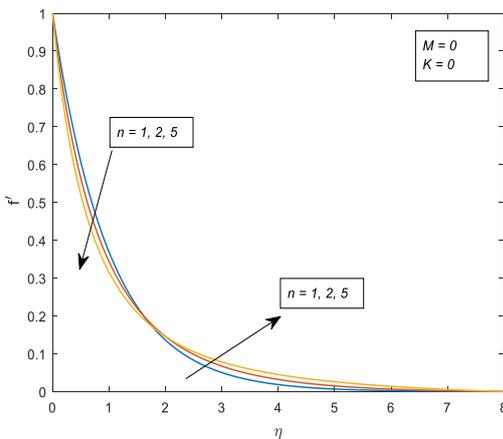


Figure 3. Variation of velocity profile  $f'$  for different values of  $n$  with  $M = 0$  and  $K = 0$

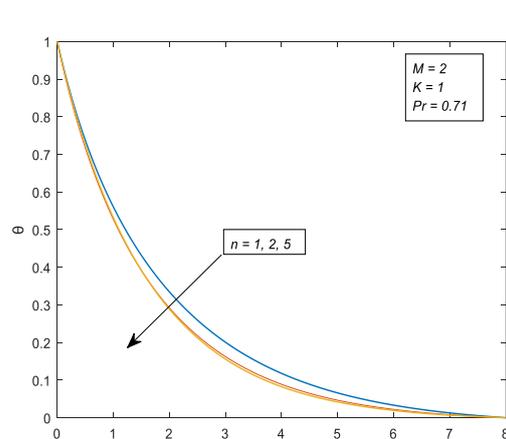
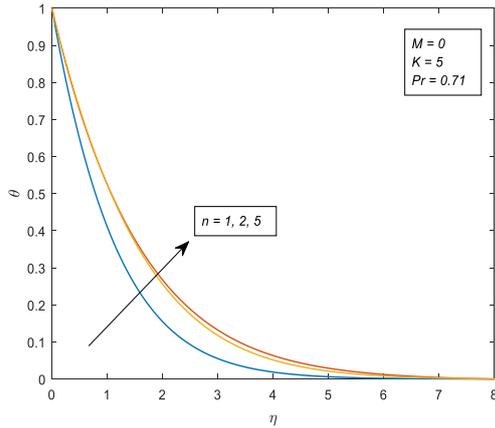
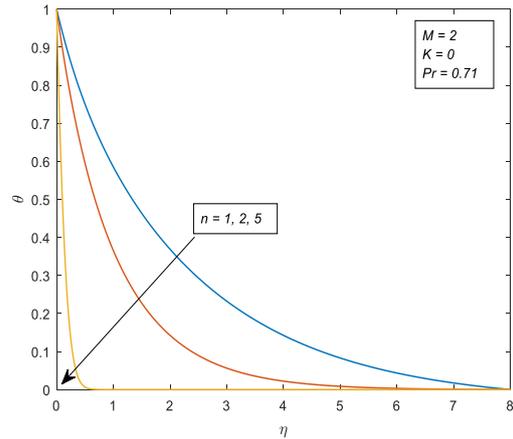


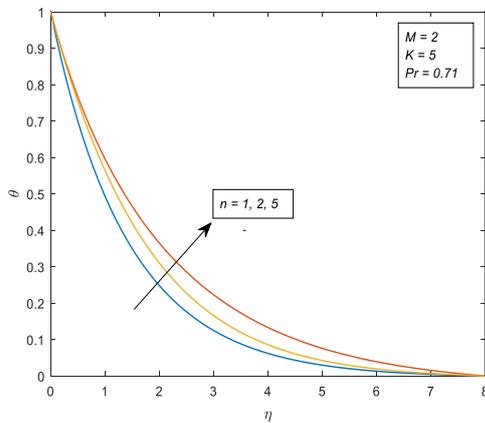
Figure 6. Variation of temperature profile  $\theta$  for different values of  $n$  with  $M = 2$ ,  $K = 1$  and  $Pr = 0.71$



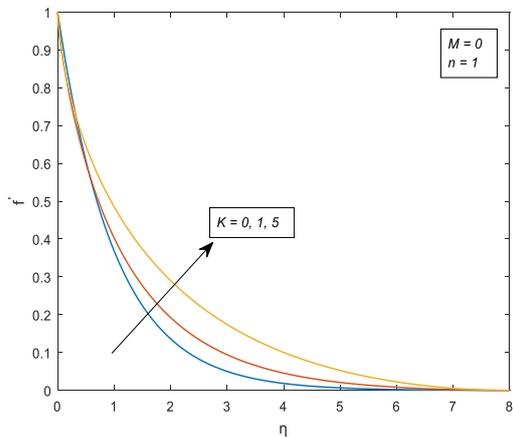
**Figure 7.** Variation of temperature profile  $\theta$  for different values of  $n$  with  $M = 0, K = 5$  and  $Pr = 0.71$



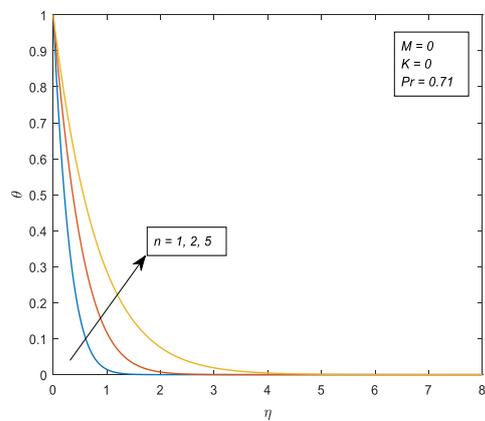
**Figure 10.** Variation of temperature profile  $\theta$  for different values of  $n$  with  $M = 2, K = 0$  and  $Pr = 0.71$



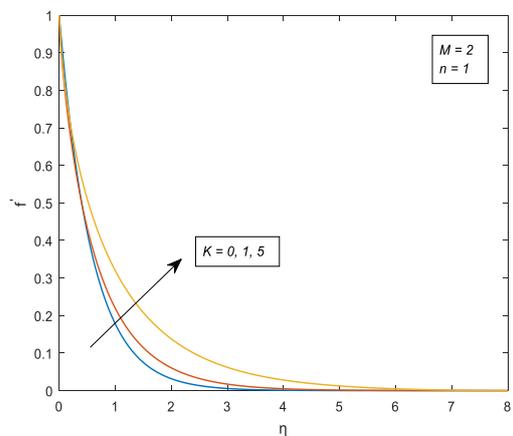
**Figure 8.** Variation of temperature profile  $\theta$  for different values of  $n$  with  $M = 2, K = 5$  and  $Pr = 0.71$



**Figure 11.** Variation velocity profile of  $f'$  for different values of  $K$  with  $M = 0$  and  $n = 1$



**Figure 9.** Variation of temperature profile  $\theta$  for different values of  $n$  with  $M = 0, K = 0$  and  $Pr = 0.71$



**Figure 12.** Variation of velocity profile  $f'$  for different values of  $K$  with  $M = 2$  and  $n = 1$

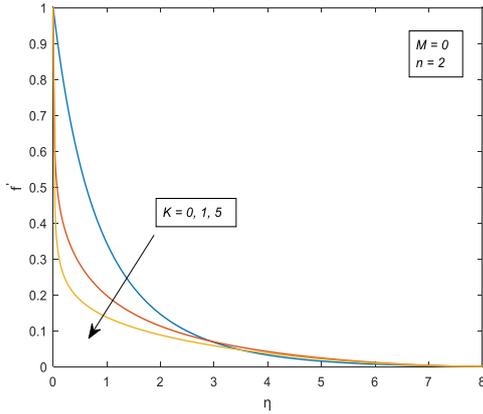


Figure 13. Variation of velocity profile  $f'$  for different values of  $K$  with  $M = 0$  and  $n = 2$

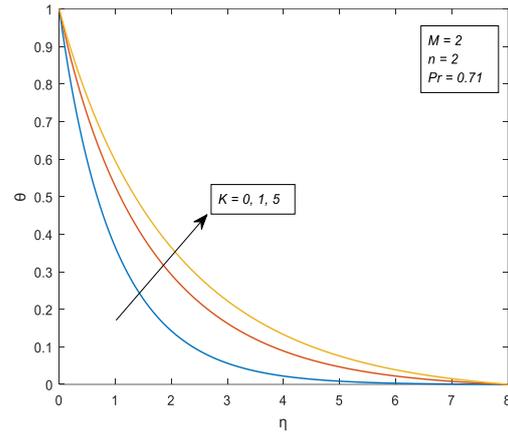


Figure 16. Variation of temperature profile  $\theta$  for different values of  $K$  with  $M = 2, n = 2$  and  $Pr = 0.71$

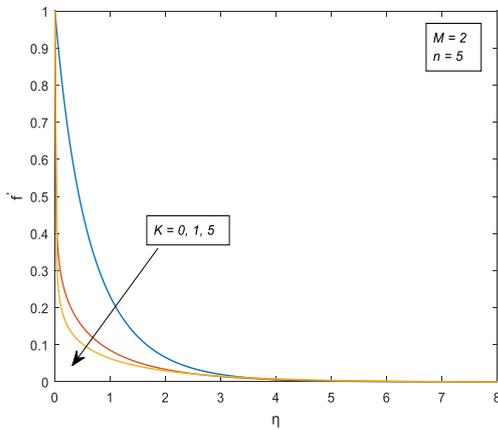


Figure 14. Variation of velocity profile  $f'$  for different values of  $K$  with  $M = 2$  and  $n = 5$

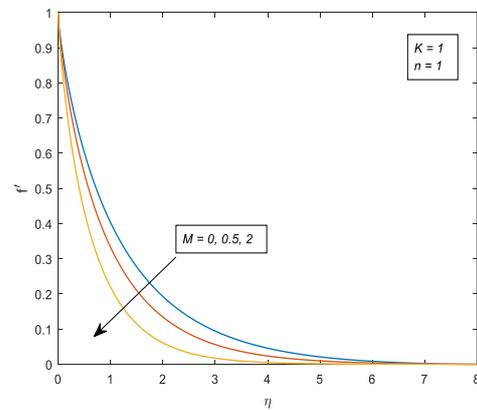


Figure 17. Variation of velocity profile  $f'$  for different values of  $M$  with  $K = 1$  and  $n = 1$

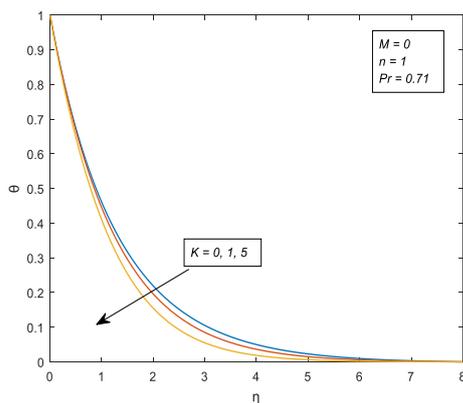


Figure 15. Variation of temperature profile  $\theta$  for different values of  $K$  with  $M = 0, n = 1$  and  $Pr = 0.71$

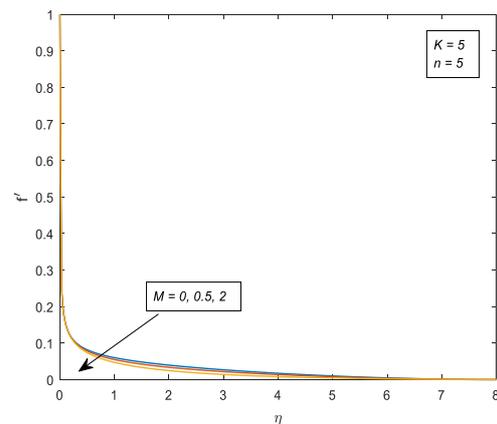
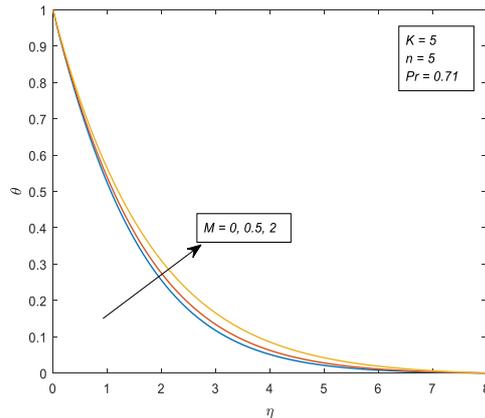


Figure 18. Variation of velocity profile  $f'$  for different values of  $M$  with  $K = 5$  and  $n = 5$



**Figure 19.** Variation of temperature profile  $\theta$  for different values of  $M$  with  $K = 5, n = 5$  and  $Pr = 0.71$

## 5. Conclusion

Viscoelastic fluid flow and heat transfer past a nonlinear stretching sheet in the presence of magnetic field have been studied and analyzed in this paper. Numerical solutions for the viscoelastic fluid problem are obtained with help of Keller box method. However, the behaviour of nonlinear stretching sheet, magnetic field and viscoelastic parameter on fluid flow and heat transfer of viscoelastic fluid over a nonlinear stretching sheet was analyzed. The following conclusion are drawn from the studies:

- i. An increase in the value of nonlinear stretching parameter decreases the velocity field and increases the temperature distributions.
- ii. It is noted that the impact of magnetic parameter is higher in temperature distributions as compared to the velocity field.
- iii. An increase in the value of viscoelastic parameter in the presence of magnetic parameter enhances the velocity profile and depreciate the temperature distributions.
- iv. The magnitude of the coefficient of skin friction increases with the increase in nonlinear stretching parameter, whereas the reverse is the case for the local Nusselt number.

## Conflict of interest

The authors declare no conflict of interest.

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