Caliphate Journal of Science & Technology (CaJoST)

ISSN: 2705-313X (PRINT); 2705-3121 (ONLINE)

Research Article

Article Info

Received: 7th June 2020

Revised: 18th July 2020

Accepted: 23rd July 2020

Local Stability Analysis of Unique Endemic Equilibrium Point of a Mathematical Model for the Transmission and Control of Zika Virus Disease Dynamics

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Cite this: *CaJoST,* 2020, 2, 175-181

A formulated and transformed model for the transmission and control of zika virus disease dynamics is showed to be mathematically well posed and suitable for analysis, by confirming the existence and uniqueness of the solution of the model and the state variables shown to be positive. The reformed model is Linearized around the endemic equilibrium point and the sub linearity trick of Krasnoselkii is used to show that the real parts of the eigenvalues of the characteristics polynomial are negative, which is a condition for the existence of locally asymptotically stability of the model. This means that the system is stable to a small perturbation of the unique endemic point of the model. The effective reproduction number that is consistently greater than one persists. With strict adherence to the various control measures (parameters) incorporated into the model, the disease can effectively be controlled in any environment, with report of zika virus disease outbreak, if the effective reproduction number is made to be less than one.

Keywords: Model, Transmission, Control, Endemic Equilibrium Point, Locally Asymptotically Stability

1. Introduction

Zika virus (ZIKV) is a microscopic infectious agent that is a member of the family called Flaviviridae and the genus called Flavivirus that causes zika virus fever in human and microcephaly (smaller head than normal) to the new borne babies of infected mothers [1]. The infection causes no or mild symptoms similar to mild form of dengue fever or malaria fever. About 80% of infectious are asymptomatic, moreover the infection rarely kills. To detect Zika virus, a blood or tissue sample from the first week of infection must be sent to an advanced laboratory so that the virus can be detected through sophisticated molecular testing. Zika virus is a native of Zika forest, Uganda where it was first discovered in monkey 1947. Zika fever has similar characteristics with Dengue and Chikungunya [2]. The four major routes of transmission of the virus are through female aedes mosquito bite [3] -[5], sexual contact [6] -[8], vertical transmission from infected pregnant women to their fetuses and mother to child transmission near delivery or at birth to a new born baby through blood transfusion [9],[10]. In [1], the unique endemic equilibrium point of the model of zika virus disease dynamics exists when the effective reproduction number is greater than one. The mathematical model of the dynamics of zika virus disease is used for the local stability analysis of the unique endemic point. The analysis provides a condition for the existence of locally asymptotically stability of the model. This means that the system is stable to a small perturbation of the unique endemic point of the model. The effective reproduction number that is consistently greater than one persists. With strict adherence to the various control measures (parameters) incorporated into the model, the disease can effectively be controlled in any environment with report of zika virus disease outbreak if the effective reproduction number is reduced to less than one; this is achieved when the sensitive parameters values are substituted into the effective reproduction number [11]. In formulating a mathematical model of the dynamics of zika virus disease, human population is split into female population and the male population, each subpopulation is further partitioned into the susceptible compartment, exposed compartment, symptomatic infectious compartment, asymptomatic infectious compartment and the

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removed compartment respectively. The mosquito population is split into the compartment of mosquitoes without zika virus and mosquitoes with zika virus.

2. Model Formulation

As in [1], the population of the study in a non enzootic region involves humans and mosquitoes respectively. The human population is subdivided into two main groups of female compartments and male compartments. Based on the interpretation of transmission and control of Zika virus, SEIR framework (Susceptible – Expose – Infected – Removed) is used for the human compartments while SI framework (Susceptible – Infectious) is used for the mosquito compartments

2.1 Model Equations

$$\dot{S}_1 = \theta_1\omega_1\Lambda_1 - \frac{S_1}{N_1}\{\alpha_1\phi_3 I_3 + \alpha_{21}\phi_{21}I_{21}(1-\epsilon_c\ \tau_c) + \alpha_{22}\phi_{22}I_{22}(1-\epsilon_c\ \tau_c)\} - \mu_1 S_1 \quad (1)$$

$$\dot{E}_1 = (1-\theta_1)\omega_1\Lambda_1 + \frac{S_1}{N_1}\{\alpha_1\phi_3 I_3 + \alpha_{21}\phi_{21}I_{21}(1-\epsilon_c\ \tau_c) + \alpha_{22}\phi_{22}I_{22}(1-\epsilon_c\ \tau_c)\} - (\gamma_1 + \sigma_{11} + \sigma_{12} + \mu_1)E_1 \quad (2)$$

$$\dot{I}_{11} = \sigma_{11}E_1 - (\gamma_{11} + \mu_1)I_{11} \quad (3)$$

$$\dot{I}_{12} = \sigma_{12}E_1 - (\gamma_{12} + \mu_1)I_{12} \quad (4)$$

$$\dot{R}_1 = (1-\omega_1)\Lambda_1 + \gamma_1 E_1 + \gamma_{11}I_{11} + \gamma_{12}I_{12} - \mu_1 R_1 \quad (5)$$

$$\dot{S}_3 = \Lambda_3 - \frac{S_3}{N_3}(\alpha_3^{11}\phi_{11}I_{11} + \alpha_3^{12}\phi_{12}I_{12} + \alpha_3^{21}\phi_{21}I_{21} + \alpha_3^{22}\phi_{22}I_{22}) - (\mu_3 + \delta)S_3 \quad (6)$$

$$\mathrm{I}_3 = \frac{S_3}{N_3}(\alpha_3^{11}\phi_{11}I_{11} + \alpha_3^{12}\phi_{12}I_{12} + \alpha_3^{21}\phi_{21}I_{21} + \alpha_3^{22}\phi_{22}I_{22}) - (\mu_3 + \delta)\,\mathrm{I}_3 \quad (7)$$

$$\dot{S}_2 = \theta_2\omega_2\Lambda_2 - \frac{S_2}{N_2}\{\alpha_2\phi_3 I_3 + \alpha_{11}\phi_{11}I_{11}(1-\epsilon_c\ \tau_c) + \alpha_{12}\phi_{12}I_{12}(1-\epsilon_c\ \tau_c)\} - \mu_2 S_2 \quad (8)$$

$$\dot{E}_2 = (1-\theta_2)\omega_2\Lambda_2 + \frac{S_2}{N_2}\{\alpha_2\phi_3 I_3 + \alpha_{11}\phi_{11}I_{11}(1-\epsilon_c\ \tau_c) + \alpha_{12}\phi_{12}I_{12}(1-\epsilon_c\ \tau_c)\} - (\gamma_2 + \sigma_{21} + \sigma_{22} + \mu_2)E_2 \quad (9)$$

$$\dot{I}_{21} = \sigma_{21}E_2 - (\gamma_{21} + \mu_2)I_{21} \quad (10)$$

$$\dot{I}_{22} = \sigma_{22}E_2 - (\gamma_{22} + \mu_2)I_{22} \quad (11)$$

$$\dot{R}_2 = (1-\omega_2)\Lambda_2 + \gamma_2 E_2 + \gamma_{21}I_{21} + \gamma_{22}I_{22} - \mu_2 R_2 \quad (12)$$

2.2 Model Variables and Parameters

The state variables and parameters of the model equations (1) – (12) are defined in Table 1 and Table 2.

Table 1: Description of variables of the model

Variable	Interpretation
S_1, S_2	Susceptible female, male compartment
E_1, E_2	Exposed female, male compartment
I_{11}, I_{21}	Symptomatic female, male compartment
I_{12}, I_{22}	Asymptomatic female, male compartment
R_1, R_2	Removed female, male compartment
S_3, I_3	Mosquitoes without virus, mosquitoes with virus

Table 2: Description of parameters of system (1) – (12)

Parameter	Interpretation
$\Lambda_1, \Lambda_2, \Lambda_3$	Recruitment rate of females, males and mosquitoes
ω_1, ω_2	Proportion of female and male births without microcephaly
$(1-\omega_1), (1-\omega_2)$	Proportion of female and male births with microcephaly
θ_1, θ_2	Proportion of susceptible female and male births without microcephaly
$(1-\theta_1), (1-\theta_2)$	Proportion of exposed female and male births without microcephaly
μ_1, μ_2, μ_3	Natural death rate of females, males and mosquitoes
δ	Death rate of mosquitoes due to insecticides

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α_1, α_2	Transmission rate of ZIKV through mosquito bite to the susceptible females and males
$\alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}$	Transmission rate of ZIKV through sex from symptomatic and asymptomatic female and males to susceptible females and males
ϕ_3	Is measuring the reduction in effectiveness of mosquito activities in transmitting ZIKV by creating non conducive environment for the mosquitoes through the use of air conditioner
$\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}$	Is measuring the reduction in effectiveness of sexual transmission through adherence to the preventive instructions
$(1-\in_c \tau_c)$	Reflects the impact of condom usage which is enhanced by public campaign (efficacy and compliance) on sexual transmission where $0 < \in_c, \tau_c < 1$
$\alpha_3^{11}, \alpha_3^{12}, \alpha_3^{21}, \alpha_3^{22}$	Transmission rate of ZIKV from symptomatic and asymptomatic females and males to susceptible the susceptible mosquitoes
$\sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}$	Progression rate of ZIKV from exposed females and males to the symptomatic and asymptomatic compartment
γ_1, γ_2	Recovery rate from exposed females and males to the removed compartment
$\gamma_{11}, \gamma_{12}, \gamma_{21}, \gamma_{22}$	Recovery rate from symptomatic and asymptomatic females and males to the removed compartment

2.3 The Transformed Model

$$\dot{S}_1 = \theta_1\omega_1\Lambda_1 - S_1(\overline{\beta_1}I_3 + \overline{\beta_2}I_{21} + \overline{\beta_3}I_{22}) - \mu_1 S_1 \quad (13)$$

$$\dot{E}_1 = (1-\theta_1)\omega_1\Lambda_1 + S_1(\overline{\beta_1}I_3 + \overline{\beta_2}I_{21} + \overline{\beta_3}I_{22}) - k_1 E_1 \quad (14)$$

$$\dot{I}_{11} = \sigma_{11}E_1 - k_2 I_{11} \quad (15)$$

$$\dot{I}_{12} = \sigma_{12}E_1 - k_3 I_{12} \quad (16)$$

$$\dot{R}_1 = (1-\omega_1)\Lambda_1 + \gamma_1 E_1 + \gamma_{11}I_{11} + \gamma_{12}I_{12} - \mu_1 R_1 \quad (17)$$

$$\dot{S}_3 = \Lambda_3 - S_3(\overline{\beta_4}I_{11} + \overline{\beta_5}I_{12} + \overline{\beta_6}I_{21} + \overline{\beta_7}I_{22}) - k_4 S_3 \quad (18)$$

$$\dot{I}_3 = S_3(\overline{\beta_4}I_{11} + \overline{\beta_5}I_{12} + \overline{\beta_6}I_{21} + \overline{\beta_7}I_{22}) - k_4 I_3 \quad (19)$$

$$\dot{S}_2 = \theta_2\omega_2\Lambda_2 - S_2(\overline{\beta_8}I_3 + \overline{\beta_9}I_{11} + \overline{\beta_{10}}I_{12}) - \mu_2 S_2 \quad (20)$$

$$\dot{E}_2 = (1-\theta_2)\omega_2\Lambda_2 + S_2(\overline{\beta_8}I_3 + \overline{\beta_9}I_{11} + \overline{\beta_{10}}I_{12}) - k_5 E_2 \quad (21)$$

$$\dot{I}_{21} = \sigma_{21}E_2 - k_6 I_{21} \quad (22)$$

$$\dot{I}_{22} = \sigma_{22}E_2 - k_7 I_{22} \quad (23)$$

$$\dot{R}_2 = (1-\omega_2)\Lambda_2 + \gamma_2 E_2 + \gamma_{21}I_{21} + \gamma_{22}I_{22} - \mu_2 R_2 \quad (24)$$

$$\left.\begin{array}{c} N_1 = S_1 + E_1 + I_{11} + I_{12} + R_1 \\ N_3 = S_3 + I_3 \\ N_2 = S_2 + E_2 + I_{21} + I_{22} + R_2 \end{array}\right] \quad (25)$$

Where

$$\left.\begin{array}{c} \overline{\beta_1} = \frac{\beta_1\mu_1}{\Lambda_1}, \overline{\beta_2} = \frac{\beta_2\mu_1}{\Lambda_1}, \overline{\beta_3} = \frac{\beta_3\mu_1}{\Lambda_1}, \overline{\beta_4} = \frac{\beta_4\mu_3}{\Lambda_3}, \overline{\beta_5} = \frac{\beta_5\mu_3}{\Lambda_3} \\ \overline{\beta_6} = \frac{\beta_6\mu_3}{\Lambda_3}, \overline{\beta_7} = \frac{\beta_7\mu_3}{\Lambda_3}, \overline{\beta_8} = \frac{\beta_8\mu_2}{\Lambda_2}, \overline{\beta_9} = \frac{\beta_9\mu_2}{\Lambda_2}, \overline{\beta_{10}} = \frac{\beta_{10}\mu_2}{\Lambda_2} \end{array}\right] \quad (26)$$

$$\left.\begin{array}{c} \beta_1 = \alpha_1\phi_3, \beta_2 = \alpha_{21}\phi_{21}(1-\in_c \tau_c), \beta_3 = \alpha_{22}\phi_{22}(1-\in_c \tau_c), \beta_4 = \alpha_3^{11}\phi_{11}, \beta_5 = \alpha_3^{12}\phi_{12}, k_6 = \gamma_{21} + \mu_2 \\ \beta_6 = \alpha_3^{21}\phi_{21}, \beta_7 = \alpha_3^{22}\phi_{22}, \beta_8 = \alpha_2\phi_3, \beta_9 = \alpha_{11}\phi_{11}(1-\in_c \tau_c), \beta_{10} = \alpha_{12}\phi_{12}(1-\in_c \tau_c), k_7 = \gamma_{22} + \mu_2 \\ k_1 = \gamma_1 + \sigma_{11} + \sigma_{12} + \mu_1, k_2 = \gamma_{11} + \mu_1, k_3 = \gamma_{12} + \mu_1, k_4 = \mu_3 + \delta, k_5 = \gamma_2 + \sigma_{21} + \sigma_{22} + \mu_2 \end{array}\right] \quad (27)$$

2.4 Basic Model Analysis

The primary aim of the model (1) – (12) is to give a mathematical interpretation and give qualitative analysis of the dynamic transmission of zika virus disease towards a good control of any outbreak of zika virus disease. Moreover, the analysis will guide health personnel, infected and non infected individuals, governmental and nongovernmental organizations to understand the dynamics and control of zika virus disease.

2.4.1 Existence and Uniqueness of Solution

For model (13) – (24) to be suitable for analysis the model must be epidemiologically meaningful and mathematically well posed. [12]. Consider the feasible region

$$D = \left\{\begin{array}{c} (S_1, E_1, I_{11}, I_{12}, R_1) \in \Re_+^5 : S_1 \ge 0, E_1 \ge 0, I_{11} \ge 0, I_{12} \ge 0, R_1 \ge 0 \\ S_1 + E_1 + I_{11} + I_{12} + R_1 \le N_1 \\ (S_3, I_3) \in \Re_+^2 : S_3 \ge 0, \ge 0, I_3 \ge 0 \\ S_3 + I_3 \le N_3 \\ (S_2, E_2, I_{21}, I_{22}, R_2) \in \Re_+^5 : S_2 \ge 0, E_2 \ge 0, I_{21} \ge 0, I_{22} \ge 0, R_2 \ge 0 \\ S_2 + E_2 + I_{21} + I_{22} + R_2 \le N_2 \end{array}\right. \quad (28)$$

Adding equations (13) through (17), (18) to (19) and (20) through (24) give:

$$\frac{dN_1}{dt} = \Lambda_1 - \mu_1 N_1 \quad (29)$$

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$$\frac{dN_3}{dt} = \Lambda_3 - (\mu_3 + \delta)N_3 \qquad (30)$$

$$\frac{dN_2}{dt} = \Lambda_2 - \mu_2 N_2 \qquad (31)$$

The sub population sizes:
$N_1(t), N_3(t), N_2(t)$ are obtained from (29), (30) and (31) as

$$N_1 \le \frac{\Lambda_1}{\mu_1}, N_3 \le \frac{\Lambda_1}{\mu_3 + \delta}, N_2 \le \frac{\Lambda_2}{\mu_2} \qquad (32)$$

$$N_1 \to \frac{\Lambda_1}{\mu_1}, N_3 \to \frac{\Lambda_3}{\mu_3 + \delta}, N_2 \to \frac{\Lambda_2}{\mu_2} \qquad as \quad t \to \infty$$

That is each subpopulation is asymptotically constant.

Theorem 1

Consider the model equations (13) to (24) with non –negative initial values

$$S_{1(0)}, E_{1(0)}, I_{11(0)}, I_{12(0)}, R_{1(0)}, S_{3(0)}, I_{3(0)}, S_{2(0)}, E_{2(0)}, I_{21(0)}, I_{22(0)}, R_{2(0)}. \qquad (33)$$

Let $D = \{(\underline{X}, t) : |X_i| \le b_0, 0 \le t < a_0\}$, where

$$\left.\begin{array}{c} \underline{X} = (S_1, E_1, I_{11}, I_{12}, R_1, S_3, I_3, S_2, E_2, I_{21}, I_{22}, R_2, N_1, N_3, N_2), \\ \theta_1, \theta_2, \omega_1, \omega_2, \Lambda_1, \Lambda_3, \Lambda_2, \mu_1, \mu_2, \sigma_{11}, \sigma_{12}, \sigma_{21}, \sigma_{22}, \gamma_1, \gamma_2, \gamma_{11}, \gamma_{12}, \\ \gamma_{21}, \gamma_{22}, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, \beta_7, \beta_8, \beta_9, \beta_{10}, k_1, k_2, k_3, k_4, k_5, k_6 \\ and \quad k_7 \end{array}\right]$$

are real positive constant and $a_0, b_0 < \infty$. Then the system (13) to (24) satisfying (33) has a unique solution which is implied by the existence of solution [13].

Proof. If equations (13) – (24) are considered in vector form with initial data

$$(S_{1(0)}, E_{1(0)}, I_{11(0)}, I_{12(0)}, R_{1(0)}, S_{3(0)}, I_{3(0)}, S_{2(0)}, E_{2(0)}, I_{21(0)}, I_{22(0)}, R_{2(0)})^T$$

Then we define equation (13)

.

$$f_i\{t, S_1(t), S_2(t), S_3(t), E_1(t), E_2(t), I_3(t), I_{11}(t), I_{12}(t), I_{21}(t), I_{22}(t), R_1(t), R_2(t)\}$$

as

$$f_1 = \theta_1\omega_1\Lambda_1 - \frac{S_1}{N_1}\{\beta_1 I_3 + \beta_2 I_{21} + \beta_3 I_{22}\} - \mu_1 S_1$$

We then obtained the following:

$$\left|\frac{\partial f_1}{\partial S_1}\right| = \left|-\frac{1}{N_1}\{\beta_1 I_3 + \beta_2 I_{21} + \beta_3 I_{22}\} - \mu_1\right| \le \left|\frac{1}{N_1}\{\beta_1 I_3 + \beta_2 I_{21} + \beta_3 I_{22}\} + \mu_1\right| = c_1 < \infty$$

$$\left|\frac{\partial f_1}{\partial I_3}\right| = \left|-\frac{S_1}{N_1}(\beta_1)\right| \le \left|\frac{S_1}{N_1}(\beta_1)\right| = C_2 < \infty, \left|\frac{\partial f_1}{\partial I_{21}}\right| = \left|-\frac{S_1}{N_1}(\beta_2)\right| \le \left|\frac{S_1}{N_1}(\beta_2)\right| = C_3 < \infty$$

$$\left|\frac{\partial f_1}{\partial I_{22}}\right| = \left|-\frac{S_1}{N_1}\beta_3\right| \le \left|\frac{S_1}{N_1}\beta_3\right| = C_4, \left|\frac{\partial f_1}{\partial E_1}\right| = \left|\frac{\partial f_1}{\partial I_{11}}\right| = \left|\frac{\partial f_1}{\partial I_{12}}\right| = \left|\frac{\partial f_1}{\partial R_1}\right| = \left|\frac{\partial f_1}{\partial S_3}\right| = \left|\frac{\partial f_1}{\partial S_2}\right| = \left|\frac{\partial f_1}{\partial E_2}\right| = \left|\frac{\partial f_1}{\partial R_2}\right| = 0$$

If equations (14) – (24) are similarly treated then we observed from the system (13) – (24) that the equalities and inequalities hold. Since $\underline{X}$ is bounded, then $f_i(\underline{X}, t), i = 1, ..., 12$ are defined and continuous for all points $(\underline{X}, t), i = 1, ..., 12$ in D. Since $f_i(\underline{X}, t)$ are continuous in D, they take their maximum in D. Let this maximum be defined

by $M'_j = \frac{\sup}{(t, \underline{X}) \in D} |f_i(t, \underline{X})|, i, j = 1, ..., 12$.

Thus $f_i(t, \underline{X})$ are defined and continuous in the region D. Then for M' such that $|f_i(t, \underline{X})| \le M'$ for all

$(t, \underline{X}) \in D$ and $\delta = \min(a_0, \frac{b_0}{M'})$, imply that

$f_i(\underline{X}, t)$ is continuous and bounded in the region D, then the model equations (13) to (24) with the initial data has a solution in the interval $|t| < \delta$.

§

2.4.2 **Positivity of the State Variables**

Lemma 1 All the solutions of the system (13) - (24) are positive for all time $t \ge 0$ if the initial conditions are positive. For the system (13) - (24), the region D is positively invariant and all solutions starting in D approach or stay in D.

Proof. Equation (13) implies

$$\frac{dS_1}{dt} \ge -(\lambda_1 + \lambda_2 + \lambda_3 + \mu_1)S_1$$

$$\Rightarrow S_1(t) = S_1(0)\exp[-\int_0^t (\lambda_1 + \lambda_2 + \lambda_3 + \mu_1)du] > 0$$

Similar approach can be used for equations (14) to (24). Therefore for all time greater or equal to zero, all the state variables are positive.

§

3. Local Stability Analysis (LAS) of the Endemic Point (ε_E)

Local stability of an endemic equilibrium point means that if there is a small perturbation to the point, then the system will move itself to the equilibrium point in some time.

Theorem 2

The associated unique endemic equilibrium point (ε_E) of the model (13) – (24) is Locally Asymptotically Stable if the real part of the eigenvalues of the characteristics polynomial of the linearized system around the endemic point are negative.

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Proof. Model (1) – (12) has an endemic equilibrium point when $R_e > 1$, [1]. Linearization of the model (13) – (24) around endemic equilibrium point (ε_E) [14], [15]

$where, S_1^{**} = N_1^{**} - E_1^{**} - I_{11}^{**} - I_{12}^{**} - R_1^{**}, S_3^{**} = N_3^{**} - I_3^{**}, S_2^{**} = N_2^{**} - E_2^{**} - I_{12}^{**} - I_{22}^{**} - R_2^{**}.$

gives

$$\left.\begin{aligned}
\dot{E}_1 &= (1-\theta_1)\omega_1\Lambda_1 + S_1^{**}(\overline{\beta_1} I_3 + \overline{\beta_2} I_{21} + \overline{\beta_3} I_{22}) - k_1 E_1\\
\dot{I}_{11} &= \sigma_{11}E_1 - k_2 I_{11}\\
\dot{I}_{12} &= \sigma_{12}E_1 - k_3 I_{12}\\
\dot{R}_1 &= (1-\omega_1)\Lambda_1 + \gamma_1 E_1 + \gamma_{11} I_{11} + \gamma_{12} I_{12} - \mu_1 R_1\\
\dot{I}_3 &= S_3^{**}(\overline{\beta_4} I_{11} + \overline{\beta_5} I_{12} + \overline{\beta_6} I_{21} + \overline{\beta_7} I_{22}) - k_4 I_3\\
\dot{E}_2 &= (1-\theta_2)\omega_2\Lambda_2 + S_2^{**}(\overline{\beta_8} I_3 + \overline{\beta_9} I_{11} + \overline{\beta_{10}} I_{12}) - k_5 E_2\\
\dot{I}_{21} &= \sigma_{21}E_2 - k_6 I_{21}\\
\dot{I}_{22} &= \sigma_{22}E_2 - k_7 I_{22}\\
\dot{R}_2 &= (1-\omega_2)\Lambda_2 + \gamma_2 E_2 + \gamma_{21} I_{21} + \gamma_{22} I_{22} - \mu_2 R_2
\end{aligned}\right] \quad (39)$$

LAS entails showing that system $z'(t) = Mf(\bar{x})z$ has no solution of the form (40) as in [14] and [15].

$$z(t) = z_0 e^{\omega t} \quad (40)$$

with

$z_0 \in C \setminus \{0\}, z_0 = (z_1, ..., z_9), z_i \in C, \omega \in C,$ and $\mathrm{Re}(\omega) \geq 0$ where C denotes complex numbers, Substituting a solution of the form (40) into the system (39) and simplifying, where

$$A = \left(\frac{S_1^{**}S_3^{**}\overline{\beta_1}\overline{\beta_4}\sigma_{11}}{k_1(k_2+\omega)(k_4+\omega)} + \frac{S_1^{**}S_3^{**}\overline{\beta_1}\overline{\beta_5}\sigma_{11}}{k_1(k_3+\omega)(k_4+\omega)}\right)z_1$$

,

$$B = \left(\frac{S_1^{**}S_3^{**}\overline{\beta_1}\overline{\beta_6}\sigma_{21}}{k_1(k_4+\omega)(k_6+\omega)} + \frac{S_1^{**}S_3^{**}\overline{\beta_1}\overline{\beta_7}\sigma_{22}}{k_1(k_4+\omega)(k_7+\omega)}\right)z_6$$

$$C = \left(\frac{S_2^{**}S_3^{**}\overline{\beta_4}\overline{\beta_8}\sigma_{11}}{k_5(k_2+\omega)(k_4+\omega)} + \frac{S_2^{**}S_3^{**}\overline{\beta_5}\overline{\beta_8}\sigma_{12}}{k_5(k_3+\omega)(k_4+\omega)}\right)z_1$$

$$D = \left(\frac{S_2^{**}S_3^{**}\overline{\beta_6}\overline{\beta_8}\sigma_{21}}{k_5(k_4+\omega)(k_6+\omega)} + \frac{S_2^{**}S_3^{**}\overline{\beta_7}\overline{\beta_8}\sigma_{22}}{k_5(k_4+\omega)(k_7+\omega)}\right)z_6$$

gives

$$\left.\begin{aligned}
(1+\frac{\omega}{k_1})z_1 &= A + B + \frac{S_1^{**}\overline{\beta_2}\sigma_{21}}{k_1(k_6+\omega)}z_6 + \frac{S_1^{**}\overline{\beta_3}\sigma_{22}}{k_1(k_7+\omega)}z_6\\
(1+\frac{\omega}{k_2})z_2 &= \frac{\sigma_{11}}{k_2}z_1\\
(1+\frac{\omega}{k_3})z_3 &= \frac{\sigma_{11}}{k_3}z_1\\
(1+\frac{\omega}{\mu_1})z_4 &= \frac{\gamma_1}{\mu_1}z_1 + \frac{\gamma_{11}\sigma_{11}}{\mu_1(k_2+\omega)}z_1 + \frac{\gamma_{12}\sigma_{12}}{\mu_1(k_3+\omega)}z_1\\
(1+\frac{\omega}{k_4})z_5 &= \frac{S_3^{**}\overline{\beta_4}\sigma_{11}}{k_4(k_2+\omega)}z_1 + \frac{S_3^{**}\overline{\beta_5}\sigma_{12}}{k_4(k_3+\omega)}z_1 + \frac{S_3^{**}\overline{\beta_6}\sigma_{21}}{k_4(k_6+\omega)}z_6 + \frac{S_3^{**}\overline{\beta_7}\sigma_{11}}{k_4(k_7+\omega)}z_6\\
(1+\frac{\omega}{k_5})z_6 &= C + D + \frac{S_2^{**}\overline{\beta_9}\sigma_{11}}{k_5(k_2+\omega)}z_1 + \frac{S_2^{**}\overline{\beta_{10}}\sigma_{12}}{k_5(k_3+\omega)}z_1\\
(1+\frac{\omega}{k_6})z_7 &= \frac{\sigma_{21}}{k_6}z_6\\
(1+\frac{\omega}{k_7})z_8 &= \frac{\sigma_{22}}{k_7}z_6\\
(1+\frac{\omega}{\mu_2})z_9 &= \frac{\gamma_2}{\mu_2}z_6 + \frac{\gamma_{21}\sigma_{21}}{\mu_2(k_6+\omega)}z_6 + \frac{\gamma_{22}\sigma_{22}}{\mu_2(k_7+\omega)}z_6
\end{aligned}\right] \quad (41)$$

The equilibrium point satisfies $\varepsilon_E = \{E_1^{**}, I_{11}^{**}, I_{12}^{**}, R_1^{**}, I_3^{**}, E_2^{**}, I_{21}^{**}, I_{22}^{**}, R_2^{**}) = M\varepsilon_E$, and the matrix M has nonnegative entries. Moreover, $(Mz)_i, i = 1, 2, ..., 9$ denotes the ith coordinates of the vector Mz.

System (41) implies

$$\left.\begin{aligned}
&[1+F_1(\omega)]z_1 = (Mz)_1, [1+F_2(\omega)]z_2 = (Mz)_2, [1+F_3(\omega)]z_3 = (Mz)_3,\\
&[1+F_4(\omega)]z_4 = (Mz)_4, [1+F_5(\omega)]z_5 = (Mz)_5, [1+F_6(\omega)]z_6 = (Mz)_6,\\
&[1+F_7(\omega)]z_7 = (Mz)_7, [1+F_8(\omega)]z_8 = (Mz)_8, [1+F_9(\omega)]z_9 = (Mz)_9
\end{aligned}\right] \quad (42)$$

If z is a solution of (42) which is in the form $[1+F_i(\omega)] = (Mz)_i$ for $i = 1, 2, ..., 9.$ then it is possible to find a minimum positive real number s, such that

$$\|z\| \leq s\varepsilon_E \quad (43)$$

We need to show that, $\mathrm{Re}(\omega) < 0$. Suppose by contradiction that, $\mathrm{Re}(\omega) \geq 0$. Case 1 if $\omega = 0$ then the linearized system is a homogeneous linear system in the variables z_i for $i = 1, 2, ..., 9.$ The determinant of this system is given by

$$\Delta = K_1K_2K_3K_4K_5K_6K_7K_8K_9(R_e - 1)$$

Thus $\Delta < 0$ if $R_e < 1$. Since determinant Δ is negative, it implies that the linearized system has a unique solution given by z = 0 which correspond to disease free equilibrium point ε_0

Case 2, $\omega \neq 0$

By assumption if $\mathrm{Re}(\omega) > 0$ then $[1+F_i(\omega)] > 1$ for all $i = 1, 2, ..., 9.$ If $F(\omega) = \min_i[1+F_i(\omega)]$ then $F(\omega) > 1$

$\Rightarrow \dfrac{s}{F(\omega)} < s.$ Since s is the minimum positive real number such that $\|z\| \leq s\varepsilon_E$, then

$$\|z\| > \frac{s}{F(\omega)}\varepsilon_E \tag{45}$$

Taking the norm of both sides of the second equation in (42) implies

$$F(\omega)\|z_2\| \le \|1+F_2(\omega)\|\|z_2\| = \|(Mz)_2\| \le M\|z_2\| \le sM(\varepsilon_E)_2 = s(\varepsilon_E)_2 = s(I_{11}^{**}) \tag{46}$$

(46) implies $\|z_2\| \le \frac{s}{F(\omega)}(I_{11}^{**})$, which contradicts (45). Hence, $\mathrm{Re}(\omega) < 0$. Thus, all eigenvalues of the characteristic equations associated with the linearized system have negative real parts which imply that the unique endemic equilibrium point (ε_E) is locally asymptotically stable whenever $R_e > 1$.

4. Conclusion

It is observed from the analysis of the epidemiologically viable and mathematically well posed model for the transmission and control of zika virus disease dynamics that the unique endemic point is locally asymptotically stable whenever the real parts of the eigen –values of the characteristic polynomial of the linearized model are negative real numbers. The epidemiological implication of the locally asymptotically stability is that if a small number of infectious individual is added or subtracted from the endemic value then it will transform into the initial point with time. This means that the system is stable to a small perturbation of the unique endemic point of the model. The effective reproduction number that is consistently greater than one remains constant. Strict adherence to the control measures incorporated into the model will control the outbreak by making the effective reproduction number to be less than one.

Acknowledgement

K. A. Adeyemo, the first author wishes to thank the Tetfund through Nigeria Police Academy, Wudil, Kano state for granting him scholarship and study fellowship during which this research was carried out.

Conflict of interest

The authors declare no conflict of interest.

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