



Article Info

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Stability Analysis of Drug Abuse and Drug – Related Crime Mathematical Model

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Drug abuse is the production, traffic and illicit use of drugs for the purpose of creating pleasurable effects on the brain. Numerous countries have been challenged with war against the distressing growth of drug abuse since it possesses morbidity, mortality and substantial socio-economic impact through its neuro-psychological effects on quality of life of the victims. Thus, in this study, a mathematical model on the transmission dynamics of drug abuse and its related crime was formulated and analyzed. The basic reproduction number R_0 for each model is calculated by using the next generation method and conditions for elimination or persistence of drug abuse and its related crime are determined. Stability analysis shows that whenever $R_0 > 1$, the drug free equilibrium is unstable. The result shows that the menace of drug abuse and its related crime can be eradicated when the reproduction number is less than unity and continue to persist when the reproduction number exceed unity.

Keywords: Drug abuse, Drug related crime, Reproduction number.

1.0 Introduction

Drug abuse is an insidious and global problem, affecting valuable human lives which had led to inestimable harm on public health, as well as social and legal issues. The United Nations Office on Drug and Crime (UNODC) defined drug abuse as illegal use of drugs which are under international controls which may or may not have illicit medical purposes but which are produced, trafficked or consumed illicitly [10]. However, effort was made on reducing the number of individuals involved in substance abuse in any community [9]. He considered the control of the spread of Marijuana smoking, one substance that is majorly abused, among adults. He proposed a deterministic model for controlling the spread of Marijuana smoking incorporating education and awareness campaign as well as rehabilitation as control measures. He formulated a fixed time optimal control problem subject to the model dynamics with the goal of finding the optimal combination of the control measures that will minimize the cost of the control efforts as well as the prevalence of marijuana smoking in a community.

Modeling illicit drug used dynamics and its optimal control analysis. The global burden of death and disability attributable to illicit drug use remains a significant threat to public health for both developed and developing nations as can be seen in [5]. They presented a new mathematical modeling framework to investigate the effects of illicit drug use in the community. In their model, the transmission process was

captured as a social “contact” process between the susceptible individuals and illicit drug users.

Analysis of a drinking epidemic model as can be seen [7]. They have developed a mathematical model of alcohol abuse which consists of four compartments corresponding to four population classes, namely, moderate and occasional drinkers, heavy drinkers, drinkers in treatment and temporarily recovered class. They discussed about basic properties of the system. Sensitivity analysis of the system was also discussed. Next,

Basic reproduction number (R_0) was calculated. Modeling drug abuse epidemics in the presence of limited rehabilitation capacity seen in [4]. Studied model limited rehabilitation through the hill function incorporated into a system of nonlinear ordinary differential equations. Not every member of the community is equally likely to embark on drug use, risk structure is included to help differentiate those more likely (high risk) to abuse drugs and those less likely (low risk) to abuse drugs.

Logit model for the determinants of drug driving in Ghana as can be seen in [6]. They investigated the use of drugs by commercial drivers in Ghana. They employed statistical concept in determining the factors that led to substances used and the various types of substances that are been used and abused by these drivers.

Formulated a model for substance (drug) abuse that explains the dynamics of the use and abuse of certain substances that are perceived as mood changer by commercial drivers seen [3]. The results showed that the contact and

imitation rates have an impact on the population of commercial drivers. There are impacts on interaction among non drug users and drug users with time in the system. An increase in the contact or imitation rate increases the population of drug users.

The deterministic evolution of illicit drug consumption within a given population as can be seen in [8]. They studied the NERA model that describes the dynamic evolution of illicit drug usage in a population. The model consists of nonusers (N) and three categories of drug users: the experimental (E) category, the recreational (R) category and the addict (A) category.

This study is designed to extract the work of [3] by incorporating drug related crime compartment and allowing individuals in both rehabilitation centers and drug related crime compartment to become susceptible.

2.0 Model Formulation

The total population at time t , denoted by $N(t)$, is sub-divided drug abuse and drug related crime into five mutually-exclusive compartments of susceptible $S(t)$, drug user $D(t)$, drug abuser $A(t)$, crime $C(t)$ and rehabilitation center $R(t)$, so that

$$N(t) = S(t) + D(t) + A(t) + C(t) + R(t) \quad (1)$$

The susceptible population, $S(t)$ comprises individuals who are at risk of using any drug or prone to crime. Individuals who use drug of any form are grouped under drug users, $D(t)$. The compartments of individuals who abuse drug of any form are classified under drug abuser, $A(t)$, $C(t)$ comprises of individuals who are influence of drug abuse to crime, $R(t)$ comprises of individuals receiving treatment or counseling and the rate at which individual get initiated to the use of drug is λ .

Table 2.1: Descriptions of Parameters Use and nominal value

Parameters	Description	Nominal Value	References
π	Recruitment rate	1.701	[9]
θ	Death due to drug abuse	0.085	[9]
r	Death due to drug use	0.04	[1]
ψ	Progression rate from drug user to drug abuser	0.1 [0.005, 0.205]	[9]
γ	Progression rate from drug abuser to crime	0.8	Assumed
μ	Natural death rate	0.05	[1]
τ	Noncompliance with the rehabilitation counseling individual move back to crime	0.2 [0,1.0]	[9]
σ	Death rate due to crime	$5.\theta$	Assumed
β	Effective contact rate	0.45	[1]
ρ_1	The rate at which those in the rehabilitation center rehabilitated	0.6 [0.2 – 0.8]	[1]
ρ_2	Individual in crime class move back susceptible class due to total repentance	$0.01 \rho_1$	Assumed
η	The rate at which crime individual are being rehabilitated	0.03	Assumed
α	Rate of peer pressure influence on drug use	0.025	[9]

Table 2.2: Description of variables

Variable	Description
$S(t)$	Susceptible Individuals
$D(t)$	Drug user
$A(t)$	Drug abuser
$C(t)$	Crime
$R(t)$	Rehabilitation center

2.1 Basic assumptions of the model

The following assumptions were made while formulating the model.

1. Death rate is not equal to birth/recruitment rate.
2. Recruited individuals are assumed to be all susceptible.
3. The model is homogeneous and depends on time t .
4. Those drug users that are both at the drug crime and rehabilitation center returned back to susceptible compartment when they stopped taking drug.

From the assumptions above, the following system of differential equations were modelled.

$$\frac{dS}{dt} = \pi - \lambda S - \mu S + \rho_1 R + \rho_2 C$$

$$\frac{dD}{dt} = \lambda S - \psi D - \mu D - rD$$

$$\frac{dA}{dt} = \psi D - \mu A - \theta A - \gamma A \tag{2}$$

$$\frac{dC}{dt} = \gamma A + \eta R - \alpha C - \mu C - \sigma C - \rho_2 C$$

$$\frac{dR}{dt} = \alpha C - \eta R - \mu R - \rho_1 R$$

Where $\lambda = \beta SD(1 + \alpha D)$

Then,
$$Ne^{\mu t} = \int \pi e^{\mu t} dt$$

$$Ne^{\mu t} = \frac{\pi}{\mu} e^{\mu t} + A$$

$$N(t) = \frac{\pi}{\mu} + Ae^{-\mu t}$$

2.2 Positivity of the Solution

Theorem: 2.1 Consider $\Omega =$

$$\left\{ (S(t) + D(t) + A(t) + C(t) + R(t)) \in R_+^5 : N \leq \frac{\pi}{\mu} \right\}$$

Then the solution set $\{S(t), D(t), A(t), C(t), R(t)\}$ are all positive for $t \geq 0$. Boundedness refers to the region in which solution of the system is bounded in $\Omega \subset R_+^5$.

The population of interest at time, (t) in (2) and taking its derivative will be

$$\frac{dN}{dt} = \frac{dS}{dt} + \frac{dD}{dt} + \frac{dA}{dt} + \frac{dC}{dt} + \frac{dR}{dt} \tag{3}$$

Substituting equations (2) into (3) yield

$$\frac{dN}{dt} = \pi - \mu S - (\mu - r)D - (\mu - \phi)A - (\mu - \delta)C - \mu R \tag{4}$$

At absent of infections that's $D = A = C = R = 0$ then equation (4) become

$$\frac{dN}{dt} = \pi - \mu S \tag{5}$$

Thus, the equation is a first order differential equation and by integration it will be

This implies
$$\frac{dN}{dt} + \mu N = \pi \tag{6}$$

$$IF = e^{\int \mu dt} = e^{\mu t} + c$$

Using condition at $t = 0$ we have $A = N(0) - \frac{\pi}{\mu}$

Then,
$$N(t) \leq N(0) + \frac{\pi}{\mu} (e^{\mu t} - 1)$$

As $t \rightarrow \infty$, the size of population $N \rightarrow \frac{\pi}{\mu}$. This

implies that, $0 \leq N \leq \frac{\pi}{\mu}$ and $N(t) \leq \frac{\pi}{\mu}$

Therefore equation (2) is positively invariant and bounded.

3.0 Equilibrium State and Stability Analysis of the Model

3.1 Equilibrium Point At equilibrium points

$$\frac{dS}{dt} = \frac{dD}{dt} = \frac{dA}{dt} = \frac{dC}{dt} = \frac{dR}{dt} = 0$$

Thus, (2) becomes

$$\pi - \lambda S - \mu S + \rho_1 R + \rho_2 C = 0$$

$$\lambda S - Q_1 D = 0$$

$$\psi D - Q_2 A = 0 \tag{7}$$

$$\gamma A + \eta R - Q_3 C = 0$$

$$\tau C - Q_4 R = 0$$

Substituting $\lambda = \beta SD(1 + \alpha D)$ into (7) to have

$$\beta DS(1 + \alpha D) - Q_1 D = 0$$

$$[\beta S(1 + \alpha D) - Q_1]D = 0 \tag{8}$$

So from (8), either

$$D = 0 \tag{9a}$$

Or

$$S = \frac{Q_1}{\beta(1 + \alpha D)} \tag{9b}$$

Where $Q_1 = \psi + \mu + r$, $Q_2 = \mu + \theta + \gamma$,
 $Q_3 = \tau + \mu + r + \rho_2$, $Q_4 = \eta + \mu + \rho_1$

3.2 Existence of Drug Free Equilibrium

Let E_0 denotes the disease free equilibrium.

Then the components of disease free equilibrium are given by

$$E_0 = (S_*, D_*, A_*, C_*, R_*) = \left(\frac{\pi}{\mu}, 0, 0, 0, 0\right)$$

3.3 Reproduction Number

The stability of E_0 can be established using the next generation matrix approach outlined in [2], such that the matrices f_i and V_i denote the new infection terms and the transfer terms are respectively given by

$$f_i = \begin{bmatrix} \beta S D (1 + \alpha D) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and

$$V_i = \begin{bmatrix} Q_1 D & 0 & 0 & 0 \\ -\psi D & Q_2 A & 0 & 0 \\ 0 & -\gamma A & Q_3 C & \eta R \\ 0 & 0 & -\tau C & Q_4 R \end{bmatrix} \tag{10}$$

Then the basic abuse number is given by

$$R_0 = \frac{\beta \pi}{\mu Q_1}$$

3.4 Local Stability of Drug Free Equilibrium

Theorem 3.1: The disease free equilibrium of (2) is locally asymptotically stable if

Proof: The jacobian matrix of (2) at E_0 is

$$J(E_0) = \begin{bmatrix} -\mu & -\beta S_* & 0 & \rho_2 & \rho_1 \\ 0 & \beta S_* - Q_1 & 0 & 0 & 0 \\ 0 & \psi & -Q_2 & 0 & 0 \\ 0 & 0 & \gamma & -Q_3 & \eta \\ 0 & 0 & 0 & \tau & -Q_4 \end{bmatrix}$$

Then the characteristics polynomial of (2) is

$$\chi^5 + a_4 \chi^4 + a_3 \chi^3 + a_2 \chi^2 + a_1 \chi + a_0 \tag{12}$$

Where

$$a_4 = (Q_1 - \beta S_*) + Q_2 + Q_3 + Q_4 + \mu$$

$$a_3 = \mu(Q_2 + Q_3 + Q_4) + Q_2(Q_3 + Q_4) + (Q_3 Q_4 - \eta \tau) + (Q_1 - \beta S_*)(Q_2 + Q_3 + Q_4 + \mu)$$

$$a_2 = (Q_3 Q_4 - \eta \tau)(\mu + Q_2) + \mu Q_2(Q_3 + Q_4) + (Q_1 - \beta S_*)[\mu(Q_2 + Q_3 + Q_4) + Q_2(Q_3 + Q_4) + (Q_3 Q_4 - \eta \tau)] \tag{13}$$

$$a_1 = \mu Q_2(Q_3 Q_4 - \eta \tau) + (Q_1 - \beta S_*)[\mu Q_2(Q_3 + Q_4) + (\mu + Q_2)(Q_3 Q_4 - \eta \tau)]$$

$$a_0 = \mu Q_2(Q_3 Q_4 - \eta \tau)(Q_1 - \beta S_*)$$

To express (13) in terms of R_0 as

$$a_4 = \mu + Q_2 + Q_3 + Q_4 + Q_1(1 - R_0)$$

$$a_3 = \mu(Q_2 + Q_3 + Q_4) + Q_2(Q_3 + Q_4) + (Q_3 Q_4 - \eta \tau) + Q_1(1 - R_0)(Q_2 + Q_3 + Q_4 + \mu)$$

$$a_2 = (Q_3Q_4 - \eta\tau)(\mu + Q_2) + \mu Q_2(Q_3 + Q_4) + Q_1(1 - R_0) [\beta Q_1 Q_2 Q_3 Q_4 \psi \gamma (\rho_1 \tau + \rho_2 Q_4) + \alpha Q_2 Q_3 Q_4 \eta \tau] \tag{13}$$

$$a_3 = \mu Q_2(Q_3Q_4 - \eta\tau) + Q_1(1 - R_0) [\mu Q_2(Q_3 + Q_4) + \beta Q_1 Q_2 Q_3 Q_4 \psi \gamma (\rho_1 \tau + \rho_2 Q_4) + \alpha Q_2 Q_3 Q_4 \eta \tau] \tag{16}$$

$$a_1 = \mu Q_2(Q_3Q_4 - \eta\tau) + Q_1(1 - R_0) [\mu Q_2(Q_3 + Q_4) + \beta Q_1 Q_2 Q_3 Q_4 \psi \gamma (\rho_1 \tau + \rho_2 Q_4) + \alpha Q_2 Q_3 Q_4 \eta \tau] \tag{15}$$

$$a_0 = \mu Q_2(Q_3Q_4 - \eta\tau) Q_1(1 - R_0)$$

Where $Q_5 = Q_3Q_4 - \eta\tau$

By Routh – Hurwitz criterion, (2) is locally asymptotically stable if all the eigenvalues (χ) of (12) has negative real part. It follows that for χ to have negative real parts, $a_i > 0$ ($i = 0, \dots, 4$) then it is clear that $a_i > 0$ for all $i = 0, \dots, 4$ if $R_0 < 1$. Hence (2) is locally asymptotically stable at E_0 if $R_0 < 1$ which completes the proof.

3.6 Local Stability of Endemic Equilibrium at Special Case

The local stability of E_1 for the equations (2) is explored for $\alpha = 0$. Thus

$$\lambda_{i, \alpha=0} = \chi_i = \beta S D \tag{17}$$

and (15) becomes

$$c_1 D + b_0 = 0 \tag{18}$$

3.5 Existence of Endemic Equilibrium

Let E_1 represent the endemic equilibrium (that is an equilibrium corresponding to a situation where drug abuse and its related crime exist). So, $D \neq 0$

Where $c_1 = \beta Q_4 [Q_1 Q_2 Q_3 - \psi \gamma (\rho_1 \tau + \rho_2 Q_4)]$
So,

$$\pi Q_4 - \frac{(\mu + \beta D + \alpha \beta D^2) Q_4 Q_1}{\beta(1 + \alpha D)} + \frac{(\rho_1 \tau + \rho_2 Q_4) Q_4 \psi \gamma D}{Q_2 Q_5} = \frac{-b_0}{D} > 0 \text{ for } R_0 > 1$$

Theorem 3.2: The endemic equilibrium of the

equations (2) for $\alpha = 0$ and $D \neq 0$ is locally asymptotically stable if $R_0 > 1$ and unstable if

$$\pi Q_2 Q_4 Q_5 \beta + \alpha \pi \beta Q_2 Q_4 Q_3 D - \mu Q_1 Q_2 Q_4 Q_5 - \beta D Q_1 Q_2 Q_4 Q_5 - \alpha \beta D^2 Q_1 Q_2 Q_4 Q_5 + \beta Q_4 \psi \gamma (\rho_1 \tau + \rho_2 Q_4) D + \alpha \beta \psi \gamma Q_4 D^2 (\rho_1 \tau + \rho_2 Q_4) = 0 \tag{14}$$

Proof: The jacobian matrix of model (2) at E_1 is given by

Rewritten (14) as

$$b_2 D^2 + b_1 D + b_0 = 0 \tag{15}$$

Where

$$b_2 = \alpha \beta Q_4 [Q_1 Q_2 Q_3 - \psi \gamma (\rho_1 \tau + \rho_2 Q_4)]$$

$$J(E_1) = \begin{bmatrix} -(\chi_4 + \mu) & -\beta S^0 & 0 & \rho_2 & \rho_1 \\ \chi_4 & \beta S^0 - Q_1 & 0 & 0 & 0 \\ 0 & \psi & -Q_2 & 0 & 0 \\ 0 & 0 & \gamma & -Q_3 & \eta \\ 0 & 0 & 0 & \tau & -Q_4 \end{bmatrix} \tag{19}$$

The characteristic polynomial of (19) is expressed as

$$\chi^5 + e_4\chi^4 + e_3\chi^3 + e_2\chi^2 + e_1\chi + e_0 \tag{20}$$

Where

$$e_4 = Q_1 + Q_2 + Q_3 + Q_4 + \mu - \beta S_0$$

$$e_3 = \lambda(Q_1 + Q_2 + Q_3 + Q_4) + \mu Q_2 + (Q_2 + \mu)(Q_3 + Q_4) + (Q_3 Q_4 - \eta\tau) + (\mu + Q_2 + Q_3 + Q_4)(Q_1 - \beta S_0)$$

$$e_2 = (Q_3 Q_4 - \eta\tau) [Q_2 + \mu + \lambda] + \lambda [Q_1(Q_2 + Q_3 + Q_4) + Q_2(Q_3 + Q_4)] \tag{22}$$

$$e_1 = \mu Q_1 Q_2 (Q_3 + Q_4) + \lambda [(Q_1 + Q_2)(Q_3 Q_4 + \eta\tau) + Q_1 Q_2 (Q_3 + Q_4)]$$

$$e_0 = \mu Q_1 Q_2 (Q_3 Q_4 - \eta\tau) + \lambda \{ Q_1 Q_2 (Q_3 Q_4 - \eta\tau) - \gamma \psi (\rho_2 Q_4 + \rho_1 Q_3) \}$$

Thus, with (20) are clearly seen to be positive. Hence the model equations (2) at $\alpha = 0$ is locally asymptotically stable at E_1 if $R_0 > 1$

4.0 Numerical Simulation and Discussion of Results

$$e_2 = [(Q_3 Q_4 - \eta\tau) + \mu(Q_2 + Q_3 + Q_4) + Q_2(Q_3 + Q_4)] [Q_2 + \mu + \lambda]$$

$$+ \lambda [Q_1(Q_2 + Q_3 + Q_4) + Q_2(Q_3 + Q_4)] \tag{21}$$

$$e_1 = \mu Q_1 Q_2 (Q_3 + Q_4) + \lambda [(Q_1 + Q_2)(Q_3 Q_4 - \eta\tau) + Q_1 Q_2 (Q_3 + Q_4)] + \mu Q_1 Q_2 (Q_3 + Q_4)$$

$$+ \mu Q_2 (Q_3 + Q_4)] + \mu Q_1 Q_2 (Q_3 + Q_4)$$

$$e_0 = \mu Q_2 (Q_1 - \beta S_0) [Q_3 Q_4 - \eta\tau] + \lambda \{ Q_1 Q_2 (Q_3 Q_4 - \eta\tau) - \gamma \psi (\rho_2 Q_4 + \rho_1 Q_3) \}$$

When

$$\alpha = 0 \tag{9b} \text{ becomes}$$

$$S_0 = \frac{Q_1}{\beta}$$

$$\Rightarrow Q_1 - \beta S_0 = 0 \tag{21}$$

Simplifying (18) using (19) to get

$$e_4 = Q_4 + Q_2 + Q_3$$

$$e_3 = \lambda(Q_1 + Q_2 + Q_3 + Q_4) + \mu Q_2 + (Q_2 + \mu)(Q_3 + Q_4) + (Q_3 Q_4 - \eta\tau) + (\mu + Q_2 + Q_3 + Q_4)(Q_1 - \beta S_0)$$

In this studied, qualitative analysis of the model was performed using the parameter of values stated in Table 1 while using the EADM (Elzaki Adomian Decomposition Method) for the following initial conditions $S(0)=500$, $D(0)=60$, $A(0)=40$, $C(0)=30$ and $R(0)=20$. All computation are made with the aid of Maple 18 software.

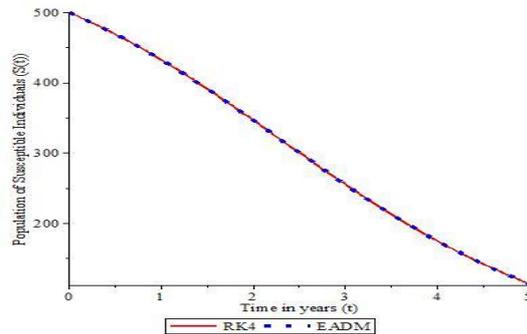


Figure 1: Graphical Comparison for Susceptible Individual S (t)

The figure above shows that the population of susceptible individuals decreases steadily with time. The steady decrease is caused by the increase in the population of drug users.

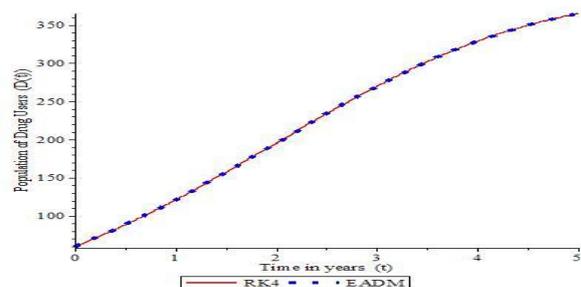


Figure 2: Graphical Comparison for Drug User D (t)

The population profile of drug users displayed in the figure above shows that as time increases the population of drug user also increases. This implies that as the population of individuals in any given community increases so the need to use drug for any purpose will increase.

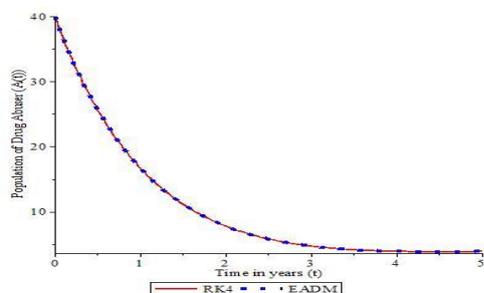


Figure 3: Graphical Comparison for Drug Abuser A(t)

Show that the number of drug abuser decreases steadily with time. This is due to the impact of effective rehabilitating drug abusers and the sensitization of the populace of the danger of drug abuse

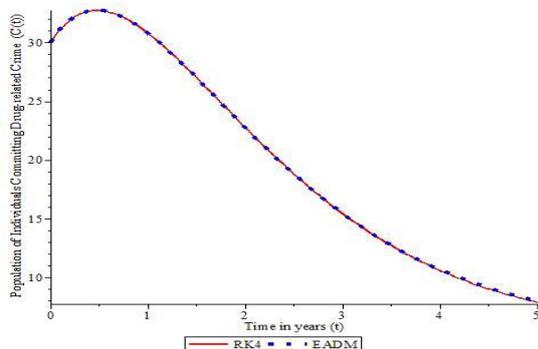


Figure 4: Graphical Comparison for Individual Committing Drug Related Crime C (t)

Initially, the population of individuals committing drug related crime increases and later decreases with time as shown above. The late decrease is caused by effective rehabilitation and public enlightenment of the populace on the disastrous consequence of drug abuse.

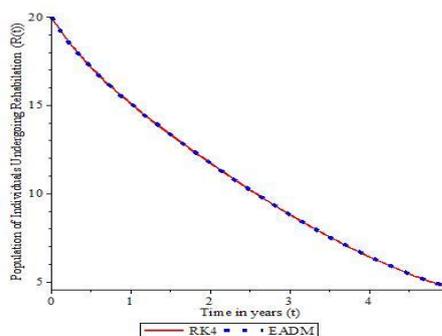


Figure 5: Graphical Comparison for Population of Individual Undergoing Rehabilitation R (t)

The figures show that population of individuals undergoing rehabilitation reduces steadily with time. As the population of drug abusers and drug related criminals who need rehabilitation will definitely decreases.

5.0 Conclusion

In this study, a five compartmental model that has been extracted as a consequence of kanyaa *et al.*, 2018 is formulated and analyzed. Some of the theoretical findings of the study are as follows

- (i) The menace of drug abuse and drug related crime can be eradicated when the reproduction number is less than unity.
- (ii) Drug abuse and drug related crime will continue to persist when the reproduction number exceed unity.

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